STAT 2290 Exam 3B, April 2026, 9 problems, 75 minutes Name: ____________________

Read these instructions:

- Leaving the testing room results in a new exam given for unfinished problems.
- Three detached sheets of notes allowed. **Turn in these notes with your exam.**
- No electronics.
- You may leave answers in terms of combinations, permutations, factorials, exponentiation, $\times$, $\div$, $+$, $-$, $\sqrt{\bullet}$ of numbers. For instance, $10 \times \sqrt{5 \times 271}/17$ is an acceptable answer.

Useful critical values and truncated R outputs:

Confidence level c	Critical value z_c
90%	1.645
95%	1.96
99%	2.575

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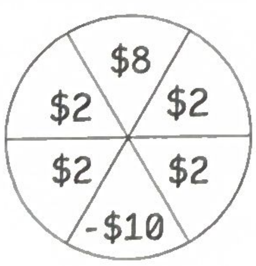
pnorm(-2) = 0.0227
pnorm(-1.5) = 0.0668
pnorm(-1) = 0.1586
pnorm(-0.5) = 0.3085
pnorm(0.5) = 0.6914
pnorm(1) = 0.8413
pnorm(1.5) = 0.9331
pnorm(2) = 0.9772
```

Problem 1. Estimate `pnorm(0, mean=3, sd=2)` from the above R outputs.

$= \text{pnorm}\left(\frac{0-3}{2}\right) = \text{pnorm}(-1.5) = 0.0668.$

Problem 2. You throw a dart, hitting the wheel below. You win or lose the monetary amount indicated by where your dart lands. Let X be the random variable modeling this monetary amount. Find the expected value and variance of X.



$X=x$	2	8	-10
$P(X=x)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$\Rightarrow E(x) = (2)\cdot\frac{2}{3} + (8)\cdot\frac{1}{6} + (-10)\cdot\frac{1}{6} = \frac{8+8-10}{6} = -1,$

$E(X^2) = (2)^2\cdot\frac{2}{3} + (8)^2\cdot\frac{1}{6} + (-10)^2\cdot\frac{1}{6} = \frac{8+32+50}{3} = 30$

$\Rightarrow \sigma^2 = E(x^2) - E(x)^2 = 30 - (-1)^2 = 29.$

Problem 3. Train passengers individually average 195 pounds with a standard deviation of 30 pounds. Passenger weights are not normally distributed but are not very skewed.

(a) Find the probability that the **average weight** of 36 randomly selected passengers is less than 190 pounds. (Justify your answer.)

Check approx normal: $n \geq 30$ ✓

Check 10% Condition: there are $N \geq 10 \cdot n = 360$ train passengers ✓

$\sigma_{\bar{x}} = 30/\sqrt{36} = 30/6 = 5$.

$\Rightarrow P(\bar{x} < 190) = \text{pnorm}(190, \text{mean}=195, \text{sd}=\cancel{36}\ 5) = \text{pnorm}\left(\frac{190-195}{\cancel{36}\ 5}\right) = \text{pnorm}(-1) = 0.1586$

(b) Find the probability that the **total weight** of 100 randomly selected passengers is less than 19800 pounds. (Justify your answer.)

First do same checks for condition: $n \geq 30$ ✓, there are $N \geq 10 \cdot 100$ passengers ✓.

avg weight $\bar{x} = \frac{19800}{100} = 198$; $P(\bar{x} < 198) = \text{pnorm}(198, \text{mean}=195, \text{sd}=\underbrace{\frac{30}{\sqrt{100}}}_{=3}) = \text{pnorm}\left(\frac{198-195}{3}\right) = \text{pnorm}(+1) = 0.8413$

Problem 4. Let $\hat{p}$ be the proportion of people in a random sample of 100 U.S. adults who say they drink the cereal milk. A spokesman for the dairy industry claims that 80% of all U.S. adults drink the cereal milk. Suppose this claim is true.

(a) What is the mean of the sampling distribution of $\hat{p}$?

$\mu_{\hat{p}} = 0.8$

(b) Find the standard deviation of the sampling distribution of $\hat{p}$. (Justify your answer.)

Check 10% Condition: there are $N \geq 10 \cdot n = 1000$ US adults ✓.

$\Rightarrow \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.8 \cdot 0.2}{100}}$.

(c) Is the sampling distribution of $\hat{p}$ approximately normal? (Justify your answer.)

Check Large Counts Condition:

$n\hat{p} = 100 \times 0.8 = 80 \geq 10$ ✓

$n(1-\hat{p}) = 100 \times 0.2 = 20 \geq 10$ ✓

So samp. dist. of $\hat{p}$ is approx normal.

Problem 5. A Pew survey found that 200 of 400 randomly selected teens reported texting with their friends every day. Calculate a 95% confidence interval for the population proportion p that would report texting with their friends every day. (Justify your answer.)

- Check 10% Cond: There are $N \geq 10 \cdot n = 10 \cdot 400 = 4000$ teens in population ✓.
- Check Large Counts: $n \cdot \hat{p} = 200 \geq 10$ ✓
  $n \cdot (1-\hat{p}) = 200 \geq 10$ ✓.
- So we can proceed:
- $c = 95\% \Rightarrow z_c = 1.96$

$$E = z_c \cdot \sqrt{\frac{p(1-p)}{n}} = 1.96\sqrt{\frac{0.5 \cdot 0.5}{400}} = 1.96 \cdot \frac{0.5}{20} \approx 2 \cdot \frac{0.5}{20} = 0.05.$$

So 95% CI is $0.5 - 0.05 < p < 0.5 + 0.05$.

Problem 6. Determine the minimum number of times you must run a Wi-Fi download speed test through your device to estimate, with 95% confidence, the average Wi-Fi download speed of your device. Your estimate must be accurate within 1 Mbps. Assume that the distribution of Wi-Fi download speeds of your device has a standard deviation of 10 Mbps.

$c = 95\% \Rightarrow z_c = 1.96$

$$1 \geq E = z_c \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{10}{\sqrt{n}} \Rightarrow \sqrt{n} \geq 19.6$$

$$\Rightarrow \boxed{n \geq (19.6)^2}$$

geometric RV, $p = 5/6$.

Problem 7. You roll a fair dice many times, and you only stop rolling once you get a non-"1" from a dice roll. Let X be the random variable that records the number of "1"s you get before you finally get a non-"1" dice roll.

(a) Determine the expected value of X.

$$E(X) = \frac{(1-p)}{p} = \frac{1/6}{5/6} = \frac{1}{5}.$$

(b) Determine the standard deviation of X.

$$\sigma^2 = \frac{(1-p)}{p^2} = \frac{1/6}{(5/6)^2} = \frac{6}{25} \Rightarrow \sigma = \frac{\sqrt{6}}{5}$$

Problem 8. The time to failure (in months) of device is a random variable whose PDF is $f(x) = c/x^3$ if $x \geq 1$ and $f(x) = 0$ if $x < 1$.

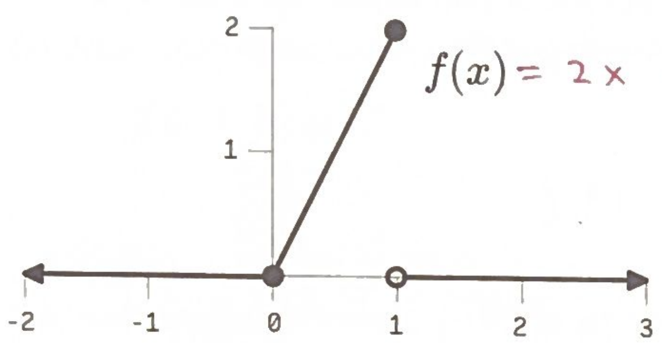
(a) Find c.

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_{1}^{\infty} \overbrace{c/x^3}^{cx^{-3}}\,dx = \left[\frac{cx^{-2}}{-2}\right]_1^{\infty} = 0 - \frac{c}{-2} \Rightarrow \boxed{c = 2}$$

(b) Find the probability that the device lasts more than 2 months.

$$\int_{2}^{\infty} f(x)\,dx = \int_{2}^{\infty} \frac{2}{x^3}\,dx = \left[-x^{-2}\right]_2^{\infty} = 0 - \left[-\frac{1}{2^2}\right] = \frac{1}{4} = \underline{25\% \text{ chance}}.$$

Problem 9. A random variable X has PDF $f(x)$ whose graph is shown:



$f(x) = 2x$ if $0 \leq x \leq 1$.

(a) Find the CDF of X.

if $0 \leq x \leq 1$, $F(x) = \int_0^x f(x)\,dx = \int_0^x 2x\,dx = x^2 \Rightarrow F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$

(b) Find the expected value and variance of X.

$$E(x) = \int_0^1 x\,f(x)\,dx = \int_0^1 x \cdot 2x\,dx = \int_0^1 2x^2\,dx = \left[\frac{2x^3}{3}\right]_0^1 = \frac{2}{3}$$

$$E(x^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 x^2 \cdot 2x\,dx = \int_0^1 2x^3\,dx = \left[\frac{x^4}{2}\right]_0^1 = \frac{1}{2}$$

$$\Rightarrow \sigma^2 = E(x^2) - E(X)^2 = \frac{1}{2} - \frac{4}{9}.$$

Page	1	2	3	4
Points	15	20	30	20
Score				